

# TURING'S THEORY OF MORPHOGENESIS APPLIED TO STREET LAYOUT, FIRST APPROACH

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**Abstract.** Urban morphology tries to understand the spatial structure of cities. It searches to identify the patterns and underlying substructures but also the process of city development. Our contribution follows this last direction. We model the development process of graphs representing the streets network using Turing model.  
**Keywords.** Graphs, Morphogenesis, Cities, Complex sociotechnical Systems

## 1 Introduction

Cities are complex sociotechnical systems [1] constituted by a very large number of heterogeneous entities which evolve and interact. Urban morphology tries to understand the spatial structure of cities. It searches to identify the patterns and underlying substructure but also the process of city development. In a first time, we consider the structure and the development of the road network that define the street layout of our towns. We know that this approach is reductionist, anyway streets give structure of our cities and we hope that the model we incrementally develop could evolve with the complexity.

The problem consist to generate graphs under constraints which are spatially and temporally coherent. The constraints influence (facilitate or impede) the development of the graph.

In 1952, Alan Turing has considered the striping patterns than we can observed for example on the tiger pelage (see figure 1). His hypothesis was that a mechanism of reaction/diffusion [2] was at the origin of the structures present in certain biological tissues. It is a chemical process with two morphogens: an activator and an inhibitor.

We revisit this model to generate graphs showing structures.

## 2 Turing Model

Turing has imagined a chemical mechanism composed of two morphogens, an activator  $A$  and an inhibitor  $I$  that diffuse and react with each other. The environment is homogeneous. The activator  $A$  favors its own formation and



Figure 1: Credit: J. Patrick Fischer / Wikimedia Commons / Public Domain

diffuses slowly. The production of the inhibitor  $I$  is stimulated by  $A$ .  $I$  diffuses rapidly and inhibits the formation of  $A$ . This mechanism generates patterns.

$$\left\{ \begin{array}{lll} \frac{\partial A}{\partial t} & = & F(A, I) \quad -d_A I \quad +D_A \Delta A \\ \frac{\partial I}{\partial t} & = & G(A, I) \quad -d_I A \quad +D_I \Delta I \end{array} \right.$$

Production    Degradation    Diffusion  
Reaction

The initial conditions are a random uniform distribution of the two morphogens.

Since  $A$  is an autocatalyst and diffuses slowly, it stimulates production of itself and it concentrates into a peak. In the same time  $A$  boosts production of its inhibitor  $I$  which diffuses more quickly than itself.  $A$  concentrations decrease around the peak. At a sufficiently large distance from the peak, the influence of the inhibitor is too low and a peak appears. Turing explains in his paper [2] that a such system yields six potential stable states, depending on the production and degradation of reaction terms and wavelength of the pattern. One of them (type 6 in the original paper) shows stationary waves with finite wavelength (see figure 2).

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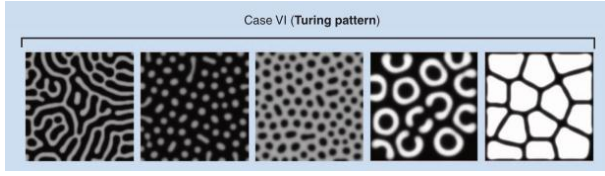


Figure 2: Two-dimensional patterns generated by the Turing model. These patterns were made by an identical equation with slightly different parameter values. Source: [3]

### 3 Coupling Turing Model and Graph

The simulation model is simple. There is a grid which represents the space where the reaction take place. The space is discretised. Each cell contains an amount of inhibitor and activator. For one step, the process is the following :

```
to Execute
  Production
  Degradation
  Diffusion
  WhatColor
end
```

The activator has a color  $C_a$  and when the activator concentration is above a threshold the cell is colored by  $C_a$ . This algorithm produces two-dimensional patterns (see figure 3).

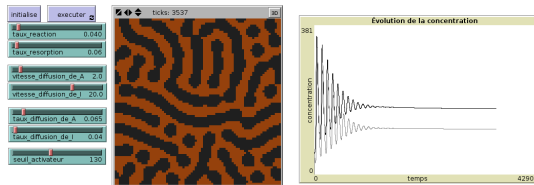


Figure 3: Netlogo simulation of Turing Model

Our model is a model with two levels. The first one is the Turing model and the second one is a temporal graph generated by the morphogenesis process (see figure 4). A node is created when the activator concentration is greater than a threshold. The creation retroacts on the first level by accelerating degradation of the activator and the production of the inhibitor. Different strategies are studied. The nodes are connected with their spatial neighbours.

### 4 Conclusion

This work is in progress. We have to study the parameter space. Anyway, we think that the concept of morphogenesis offers a new dimension to the urban study: the time,

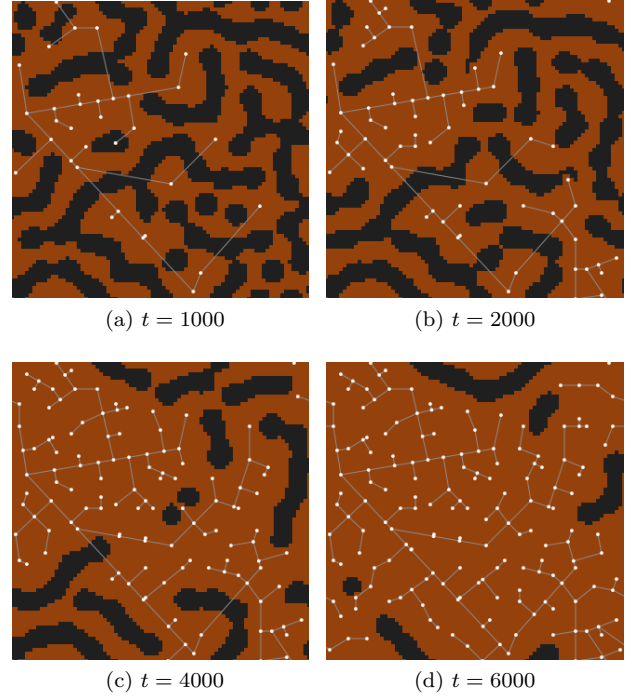


Figure 4: Example of Turing model coupled with a graph

considered here as a dynamic continuum rather than as a result of a succession of distinct states. The morphogenesis tries to capture the organizational laws underlying the existence and development of urban forms. We have also to define a third level representing the streets layout (curves, intersections ...).

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### References

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