Noname manuscript No.
(will be inserted by the editor)

A reaction-diffusion system drives the morphogenesis of spatial complex networks

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Received: November 8, 2021

1 Introduction

Complex networks have irregular architecture, they often show structures where links represent interactions or the physical support of the system. This is the case of spatial network (SN), where properties, functioning and evolution are strongly impacted by spatial features [1]. Street, leaf, insect and vascular networks are some examples of SN.

An important aspect to study is the way SNs form and evolve in time. In this paper we address the morphogenesis of SNs through a graph generator model. We start from a basic consideration: in many real situations, the morphogenesis of those networks cannot be investigated considering them as isolated, disregarding their environment. For instance, we can consider a street network as a part of an urban system. Around the street network, physical and abstract elements of the city organize themselves. Those elements interact with and through the street network, locally controlling its morphogenesis. Some elements move and their arrangement forms spatial and evolving patterns.

In our model, a set of elements, called morphogens, controls the morphogenesis of the SN [4]. Two types of decentralized and local dynamics resume the behaviour of morphogens: they move in space and they interact. Mathematically, those dynamics are expressed as a reaction diffusion (RD) system [6], where two kinds of morphogens (A and B) react and diffuse in Euclidean space. At a microscopic scale, A catalyses its own production and also the production of B. At the same time, B inhibits the production of A. B diffuses faster than A. At a macroscopic scale, the RD system can yield evolving patterns of concentration [3].

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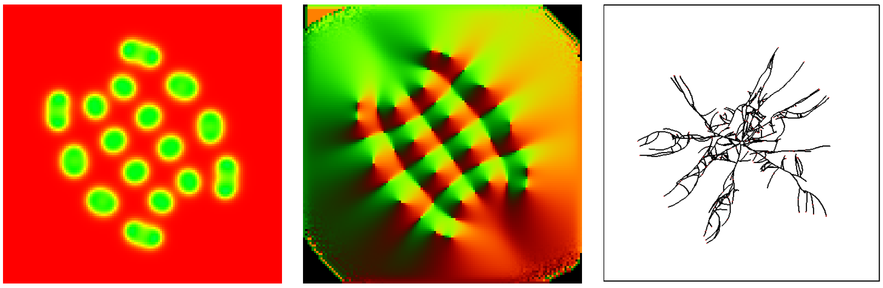

Fig. 1: The three interdependent layers of the model. The RD layer $L(t)$, the vector field layer $\vec{L}(t)$ and the dynamic geometric graph $G(x,t)$. Figures are captured after 4000 steps. Parameter set: $|L| = 2^7 \times 2^7, D_a = 0.5, Db = 2\ D_a, f = 0.030, k = 0.062$. The vector field layer is a raster image, where colours are coded with the RGB model. The magnitude V of a vector is decomposed to their two components V_x and V_y. The triples RGB associated to the corresponding pixel is $RGB = \{r = (V_x/V + 1)/2, g = (V_y/V + 1)/2, b = 0.0\}$.

2 Model description

We present a graph generator model composed by three interdependent layers (fig. 1). The first layer $L(t)$ (fig. 1, left) is a regular grid of square cells. A couple $(a,b), a \in [0.0, 1.0], b \in [0.0, 1.0]$ is assigned to each cell and indicates the concentration of morphogens A and B in the cell. Inspired by the Gray-Scott model [2], the concentrations in a cell update in accordance with the function:

$$\begin{cases} a(t+1) = a(t) + D_a\ K \times N_a(t) - a(t)b(t)^2 + f(1 - a(t)) \\ b(t+1) = b(t) + D_b\ K \times N_b(t) + a(t)b(t)^2 - (f + k)b(t) \end{cases} \tag{1}$$

where D_a and D_b are diffusion rates ($D_a \geq 2\ D_b$), K is a 3×3 bi-symmetric matrix, N_a and N_b are the concentrations of morphogens in the cells of the Moore neighbourhood associated to the cell, f and k are the feed and the kill rate.

The second layer is a dynamic vector field $\vec{L}(t)$ (fig. 1, middle). Inspired by image processing, here the convolution is a function that maps a displacement vector to each cell: $L(t) \times K \times N \rightarrow \vec{L}(t)$, where $L(t)$ is the RD layer at time step t, K is a bi-symmetric matrix, N is the neighbourhood associated to each cell.

The final layer is a dynamic geometric graph $G(x,t)$ (fig. 1, right). A couple of coordinates is associated to each vertex. To build the network, we define a set of virtual vertices named seeds, which move in space. At each time step, their position changes according to their closest vectors. A random vector diverts the deterministic movement due to the vector field. If the seed does not intersect an existing edge, it builds a geometric path graph, otherwise we remove the seed and we make an intersection of degree 3. Seeds can be removed and new seeds can be added over existing vertices. Those events are controlled by two probabilistic parameters. The result of this process is a geometric graph embedded in two-dimensional Euclidean space. At each time step, vertices are always connected and edges do not cross.

The simulation proposed in this paper starts with few connected vertices in the middle of space. The couple $(a, b) = (1.0, 0.0)$ is assigned to each cells in $L(t)$. In order to activate the generative process, cells in the middle of space are perturbed with a pulse $(a, b) = (1.0, 1.0)$. At each time step, we first update the $L(t)$ and then we compute the corresponding $\vec{L}(t)$. We eventually create new seeds and we remove some of them with two random procedures. When seeds are not removed, they move and build the network.

3 Conclusion and perspectives

Our model can be applied to simulate the morphogenesis of different SNs, such as insect nests, cracks on the surface of materials, leaf veins, vascular systems and infrastructures. In [5] the model was applied to study urban growth and we start a process of validation. It was shown that the main properties of generated graphs are similar to real street networks. In this context, morphogens can represent real activators and inhibitors of urban growth, e.g. population, economical factors and political actors. If we go through a more data-driven approach, the model can be implemented with different kinds of real data. If the simulation starts with real street network, main properties of real networks are always preserved. Our formalism allows us to integrate different vector fields which model the impacts of static spatial characteristics, e.g. the orography and artificial constraints. We think the model may be helpful both to investigate urban growth and to support decisions of urban planners. This paper does not consider the effects that emerging networks have to their morphogenetic elements. Different kinds of cross-layers feedbacks have already been modelled and we will explore in further works those possibilities.

Acknowledgements This work is supported by the project "AMED" co-funded by ERDF and the region Normandy.

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